Problems with the operator $\nabla \times (\alpha \nabla \times \ \cdot)$

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Consider the problem of finding $\boldsymbol{h}$ such that

$$\begin{cases} \nabla \times \big(\alpha\, \nabla \times \boldsymbol{h}\big) = \boldsymbol{f} \qquad \text{and} \qquad \nabla \cdot \boldsymbol{h} = 0 \qquad \text{in } \Omega, \\ \boldsymbol{h} \cdot \boldsymbol{n} = \varphi \quad \text{and} \quad \alpha(\nabla \times \boldsymbol{h}) \times \boldsymbol{n} = \boldsymbol{\omega} \times \boldsymbol{n} \quad \text{on } \Gamma, \\ \langle \boldsymbol{h} \cdot \boldsymbol{n}, 1 \rangle_{\Sigma_j} = 0, \qquad j = 1, \dots, J, \end{cases} \tag{$\mathbf{E}_{\boldsymbol{\tau}}$}$$

where Ω is a bounded open multiply connected subset of $\mathbb{R}^3$, with a smooth boundary Γ, being Σ_j, $j = 1, \dots, J$ cuts. Assuming $0 < \alpha_* \leq \alpha \leq \alpha^*$, $\boldsymbol{f}$ and the boundary conditions satisfy natural assumptions, we prove existence and uniqueness of weak and strong solutions.

The proof of existence, uniqueness and regularity of solution of problem $\mathbf{E}_{\boldsymbol{\tau}}$ depends on the study of the simpler problem $\nabla \cdot (a(x)\nabla u) = g$ with Dirichlet boundary condition. We prove existence and uniqueness of weak and strong solutions, imposing that $0 < a_* \leq a \leq a^*$ and suitable regularity to the given data. We obtain precise estimates for the norm of the solutions, depending on the norms of the data.

We use the estimates obtained to prove existence of solution of a coupled electromagnetic induction heating problem.

Work in progress with Chérif Amrouche.